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Numerical modelling of two-dimensional and axisymmetric gravity currents

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SUMMARY

A series of two-dimensional (2D), axisymmetric and three-dimensional (3D) numerical flow simulations using an implicit large Eddy simulation (ILES) are carried out for gravity-driven fluid flows. The results are compared directly with experiments undertaken to test the model. Two-dimensional results show that the Large Eddy algorithm is successful in modelling a gravity current's large scale structure. Examination of the 3D results shows that macroscopic features of the flow (the lobe and cleft instability) observed at the interface between the light and dense fluid are also modelled well. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: Implicit Large Eddy Simulation (ILES); gravity-driven flows; gravity current

1. INTRODUCTION

Gravity currents have been studied extensively in different configurations because of their importance to a large number of industrial and environmental situations. Examples of gravity currents that occur commonly include snow avalanches and ash clouds vented from volcanoes; another flow of interest is the spread of a dense gas due to the rupture of a chemical storage tank. Understanding the dynamics of the gravity current is therefore crucial for better risk assessment. Gravity currents are also relatively straightforward to examine experimentally and therefore represent a good reference test for variable-density numerical schemes.

The majority of experimental work on gravity currents [1] has been carried out in parallel-walled tanks (confined 2D channels); in such cases three distinct regimes have been identified according to the physical process that dominates the flow. However, experimental results for

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axisymmetrical gravity currents show that there are significant differences between the structure of gravity currents that propagate in 2D and those that spread axisymmetrically. Patterson *et al.* [2] have observed that in certain situations a ring vortex is generated above the front; this ring vortex can dominate the propagation of the front and change the velocity profile within the head of the gravity current. The ring vortex is able to remain stable due to the radial stretching that the head experiences as it propagates outwards; the ring vortex breaks down when the rate of stretching reduces allowing perturbations to the velocity field to destroy the coherent vortical structure.

Close inspection of a gravity current's head for both the 2D and axisymmetrical cases reveals the formation of lobe and cleft instabilities (LCI) at the head of the flow. Simpson [3] first noted the formation of lobe structures and their bifurcation through clefts in a 2D gravity current. He proposed that the formation of clefts is due to a gravitational instability related to the overrunning of less dense fluid by the dense fluid in the gravity current. The less dense fluid then forces itself upward through the gravity current causing a cleft.

Previous work on gravity currents with a significant numerical content includes that of Halworth *et al.* [4] who examined the evolution of gravity currents in rotating and non-rotating reference frames using an axisymmetric Navier–Stokes solver, the numerical results compared well with experiments. In three dimensions, Hartel *et al.* [5] have carried out direct numerical simulations (DNS) of a gravity current; the DNS yields results that are comparable with experimental observations, the Reynolds numbers are, however, too low to be of any practical interest.

In the following work we use an implicit large Eddy simulation (ILES) described in Almgren *et al.* [6] to model a gravity currents evolution. Two-dimensional, axisymmetric and 3D numerical flow simulations have been carried out and the results are compared directly with recent experimental work.

2. NUMERICAL METHOD

The equations of motion describing density-driven flows in an immiscible fluid are

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} = \frac{1}{\rho}(-\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{H}_u) \tag{1}$$

$$\rho_t + \nabla\cdot(\rho\mathbf{u}) = 0 \tag{2}$$

$$c_t + (\mathbf{u}\cdot\nabla)c = k\nabla^2 c + H_c \tag{3}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{4}$$

where $\mathbf{u}=(u,v,w),\rho,c$ and p represent the velocity, density, concentration of an advected scalar, the pressure, respectively, and $\mathbf{H}_u=(H_x,H_y,H_z)$ represents any external force. Here μ represents the dynamic viscosity, k is the diffusive coefficient for c, and H_c is the concentration source term.

The scheme is an incompressible Navier–Stokes solver where the equations of motion are solved using a second-order upwind Godunov method for the advection step, Crank–Nicolson discretization of the viscous and diffusive terms and a variable-density approximate projection to impose the divergence constraint.

The philosophy behind ILES is that it does not utilize an explicit sub-grid closure scheme applied to solve turbulence. The simulations are used to resolve the macroscopic scales in the problem and the flux-limiting monotone algorithms used for solving the set of coupled equations act in such a way as to inherently maintain physically reasonable behaviour near the grid scale cut-off. Oran and Boris [7] give a full description of the ILES methodology.

3. RESULTS

3.1. The development of a two-dimensional gravity current

The development of a full-depth gravity current is shown in Figures 1(a)–1(d), which present experimental snapshots of the flow, together with the corresponding density plots from a 2D numerical simulation, the reduced gravity $g' = \Delta\rho g/\rho$ is used to describe the initial conditions. At early times the figures show the development of the head of the gravity current (a); the front shape is predicted well at this early stage. The formation of Kelvin–Helmholtz (KH) billows (b) on the interface as the current propagates forwards is captured by the simulation, although a small phase difference exists. The subsequent 3D breakdown of the billows (c) cannot be modelled by the 2D simulation. In (d) the KH billows are still present in the numerical simulation. The front position of the bottom propagating gravity current is also somewhat retarded compared to the experimental results. The position and form of the gravity current propagating along the free surface is well predicted at this late stage.

3.2. The development of an axisymmetric gravity current

The development of a full-depth axisymmetrical gravity current can be observed in Figures 2(a)–2(d), the experiment is conducted in a wedge-shaped tank (the axis of symmetry is centred on the origin in the vertical direction) that allows the cross-sectional development of the flow to be recorded. The results are displayed in the same format as the 2D case. The axisymmetrical form of the Navier–Stokes equations are used to model this flow. The figures clearly show a frontal bulge starting to develop shortly after the lock release (a). Then during the collapse of the dense fluid volume, vorticity is generated at the interface between the lighter and denser fluids. While being advected by the radially spreading fluid, the vorticity begins to form a coherent structure near the front, which eventually grows into the leading ring vortex. A secondary ring vortex is also generated and the locations of the ring vortices are marked as white circles in (b). Vortex stretching appears to stabilize the rings (c) and these ring vortices become more intense. At a later stage the vortical flow can no longer be maintained and it vanishes abruptly. The front then continues to propagate outward without such a clearly defined structure. Comparison of the experimental and numerical results (a)–(c) shows very reasonable agreement for the initial stages of the flow (before vortex breakdown), with the front position and the form of the gravity current being accurately predicted. In the experiments a slight transverse perturbation to the flow can cause breakdown when the radial stretching becomes weak. The axisymmetrical form of the Navier–Stokes equations does not

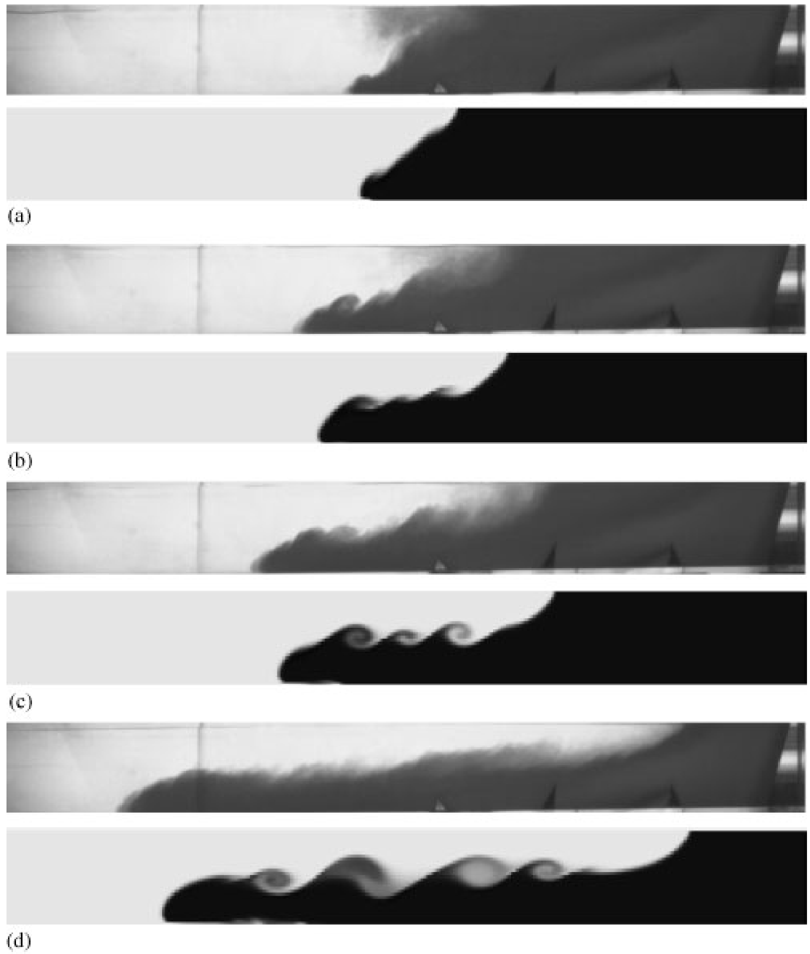

Figure 1. Experimental (upper picture) and numerical (lower picture) results showing the evolution of a gravity current in a parallel-walled tank with $g' = 4.7\ \mathrm{cm/s^2}$ for times $t = 3.13$ s (a), 5.67 s (b), 8.14 s (c), and 15.45 s (d). The numerical simulations are carried out on a grid 256×32, with a corresponding grid scale cut-off of 0.9 cm.

allow this breakdown mechanism, and in (d) (numerical result) we can see the effects of the continuing propagation of the ring vortex on the flow field.

3.3. The lobe and cleft instability

The LCI also appears to be a self-perpetuating phenomena, McElwaine *et al.* [8]; i.e. once the density or velocity field has been perturbed sufficiently flow cannot return to a uniform front condition. The LCI is important because it relates directly to the amount of entrainment of ambient fluid into the gravity current's head. Comparison of the experimental and numerical results Figures 3(a)–3(b) shows that there appears to be good qualitative agreement in the structure of the flows.

Figure 2. Experimental (upper picture) and numerical (lower picture) results showing the evolution of the radially spreading gravity current for an axisymmetric lock release with $g' = 13.2\,\text{cm/s}^2$ for times $t = 2.59\,\text{s}$ (a), 6.19 s (b), 8.70 s (c), and 12.29 (d). The numerical simulations are carried out on a grid 256×32, with a corresponding grid scale cut-off of 0.9 cm.

Close inspection of the development of the lobe and cleft structures indicates that in some simulations the formation of the first cleft structure is due to jetting of the dense fluid one grid cell from the corner boundary rather than the gravitational instability observed by Simpson [3]. Jetting of the dense fluid on the boundary can lead to a front where the edges (near the boundaries) have propagated well ahead of the front in the centre of the tank. This is a physically unrealistic front condition.

4. CONCLUSIONS

The aim of this work has been to examine at a qualitative level the suitability of an ILES for the modelling of gravity-driven flows; a more in-depth quantitative examination is underway.

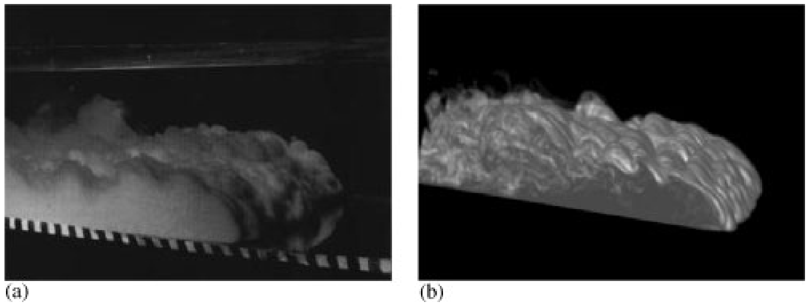

Figure 3. (a) The frontal region of a gravity current of salt water advancing along the floor of a fresh water tank; and (b) A numerical simulation of a similar reduced gravity on a $256 \times 64 \times 32$ grid using one level of adaptive mesh refinement (refining on vorticity).

The results from the 2D case show that the position and form of the gravity current is generally well predicted. The 2D simulation does not, however, allow for the breakdown of KH billows. For the axisymmetric case, the form of the gravity current's head and the ring vortices above the head are accurately described by the simulations. Good agreement is observed between the experimental work and the simulations up until the point of vortex breakdown. Simulations based on modelling the axisymmetrical form of the Navier–Stokes equations cannot match the experimental observation after the breakdown of the ring vortex in the experiments because there is no mechanism for breakdown in the equations of motion. In the experimental case azimuthal perturbations to the flow can cause breakdown when the ring vortex is weak enough.

There appears to be good qualitative agreement at a macroscopic level between the 3D numerical simulations and the flows observed experimentally. In particular, the 3D simulations allow for the breakdown of KH billows, this breakdown agrees with experimental observation and overcomes the problems experienced at later stages in the 2D simulations where the continued presence of coherent structures tend to dominate the flow field. Examination of lobe and cleft development shows that the first cleft structure appears to be formed by a numerical artefact propagated from the boundary rather than the gravitation instability observed in the experiments. Once the initial LCI has formed, the self-perpetuating nature of the features means that a physically reasonable flow field is developed. While 2D simulations of gravity currents are commonplace, few fully 3D simulations of a gravity current have been carried out to date, the results are being further analysed to examine entrainment rates and mixing efficiency in gravity-driven flows.

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